Titles and abstracts of the talks

ALKAGE workshop, Institut Fourier Grenoble, March 9 – March 13, 2020

• **Rodolfo Aguilar-Aguilar** (Institut Fourier Grenoble)

The fundamental group of quotients of products of certains topological spaces by a finite group

Abstract. We provide a generalization of a theorem of Bauer-Catanese-Grunewald-Pignatelli describing the fundamental group of a quotient of product of curves by a finite group. Our theorem applies to topological spaces admitting a universal cover and satisfying that for any $g \in G$, there is only a finite number of path-connected components in X^g, the set of points fixed by g.

• **Gergely Bérczi** (Aarhus University)

Non-reductive geometric invariant theory and hyperbolicity

Abstract. Using intersection theory on non-reductive geometric invariant theoretic quotients and work of Riedl and Yang we recently completed a proof of the Green-Griffiths-Lang and Kobayashi hyperbolicity conjectures for generic hypersurfaces of polynomial degree. We explain elements of the proof. Joint work with F. Kirwan.

• **Benoît Cadorel** (Université de Lorraine)

Generalized algebraic Morse inequalities and jet differentials

Abstract. Since their introduction by Green and Griffiths in 1979, jet differential equations have become a fundamental tool for studying entire curves on complex manifolds. The strong restrictions that these holomorphic differential equations imply on the geometry of entire curves have yielded strong hyperbolicity results for many classes of manifolds, such as high degree hypersurfaces, the complements of these hypersurfaces, surfaces of general type ... The main problem to apply these techniques is to be able to produce sufficiently many such differential equations on the manifolds under study. The most general result in this direction is due to J.-P. Demailly, who has proven that on any complex projective manifold of general type, there are "many" jet differential equations of high order. Demailly's proof is essentially analytic in nature: it relies on the use of metric methods, and in particular on his famous holomorphic Morse inequalities. We will present here an entirely algebraic proof of Demailly's theorem: the main tool in our proof will consist in the use of new algebraic Morse inequalities, generalizing in particular the other algebraic versions due to Siu, Demailly and Angelini.

• **Frédéric Campana** (Université de lorraine)

Symmetric products and fibration of compact Kähler manifolds

Abstract. If X is a compact Kähler manifold and $f : X \to B$ a fibration, it induces functorially, for any $m > 0$, a fibration $\mathrm{Sym}^m(f) : \mathrm{Sym}^m(X) \to \mathrm{Sym}^m(B)$. We show that if f is either the Iitaka, or the MRC, or the core fibration of X, then $\mathrm{Sym}^m(f)$ is the similar fibration of $\mathrm{Sym}^m(X)$ (when X is a curve, the statement needs some adaptation). In particular, X is of general type (resp. rationally connected, special) if and only if so is $\mathrm{Sym}^m(X)$ for any or some $m > 0$. The proof rests in particular on the fact that the singularities of $\mathrm{Sym}^m(X)$ are canonical, a fact due to Arapura-Archava.

This is part of a joint project (still in progress) with B. Cadorel and E. Rousseau.

• **Junyan Cao** (Institut de Mathématiques de Jussieu)

On the Ohsawa-Takegoshi extension theorem

Abstract. Since it was established, the Ohsawa-Takegoshi extension theorem turned out to be a fundamental tool in complex geometry. We establish a new extension result for twisted canonical forms defined on a hypersurface with simple normal crossings of a projective manifold with a control on its L^2 norm. It is a joint work with Mihai Păun.

• **Lionel Darondeau** (Institut Montpelliérain Alexandre Grothendieck)

Ampleness modulo the boundary of the logarithmic cotangent bundle in low degrees

Abstract. By a recent work of Brotbek and Deng, taking sufficiently many generic high degree hypersurfaces in projective spaces, one gets a logarithmic pair with "almost ample" logarithmic cotangent bundle. The minimal number of components of the boundary is half the dimension of the ambient space. Another direction of investigation is to focus on the degrees of the boundary components. In this talk, we will explain how elementary arguments allow to generalize a very nice example of Noguchi about (very) ampleness modulo the boundary of the logarithmic cotangent bundle for generic hyperplane arrangements. We will even provide a plain explicit criterion on the equations of the boundary.

This is a part of some ongoing work with Erwan Rousseau.

• **Ya Deng** (Institut des Hautes Études Scientifiques)

Big Picard theorem for varieties admitting a variation of Hodge structure

Abstract. For a quasi-projective variety U which admits a polarized variation of Hodge structures (PVHS) with quasi-finite period map, we prove that U is algebraically hyperbolic in the sense of Demailly, and that any holomorphic map from the punctured disk to U extends to a holomorphic map of the disk into any projective compactification of U. This can be seen as a generalized big Picard theorem, and it is a transcendental analogue of a previous work by A. Borel for hermitian symmetric spaces in 1972. As a consequence, we show that any holomorphic map from a quasi-projective variety to U is algebraic. This extends and strengthens a previous work by Bakker-Brunebarbe-Tsimerman, in which they use o-minimal structures to prove the algebraicity of analytic maps to the above U under the extra assumption that monodromy groups of PVHS are arithmetic. If time permits, I will discuss some open questions related to this talk.

• **Gerd Dethloff** (Université de Brest)

Normal families of meromorphic mappings with few hypersurfaces

Abstract. This talk is on joint work with Si Duc Quang (Hanoi National University of Education). We obtain some normality criteria for families of meromorphic mappings of a domain $D \subset \mathbb{C}^m$ into $\mathbb{P}^n(\mathbb{C})$ under a condition on the inverse images of only $n+2$ hypersurfaces. We get them by establishing a second main theorem without exceptional set for meromorphic mappings from a ball in $\mathbb{C}^m$ into $\mathbb{P}^n(\mathbb{C})$ intersecting hypersurfaces in general position and with error term in which the dependence of these mappings can be explicitly controlled.

• **Siarhei Finski** (Institut Fourier Grenoble)

Riemann-Roch-Grothendieck theorem for families of curves with hyperbolic cusps and its applications to moduli spaces

Abstract. We will present a refinement of Riemann-Roch-Grothendieck theorem on the level of differential forms for families of curves with hyperbolic cusps. The study of spectral properties of the Kodaira Laplacian on a Riemann surface, and more precisely of its determinant, lies in the heart of our approach.

When our result is applied directly to the moduli space of punctured stable curves, it expresses the extension of the Weil-Petersson form (as a current) to the boundary of the moduli space in terms of the first Chern form of a Hermitian line bundle. This provides a generalisation of a result of Takhtajan-Zograf.

We will also explain how our results imply some bounds on the growth of the Weil-Petersson form near the compactifying divisor of the moduli space of punctured stable curves. This would permit us to give a new approach to some well-known results of Wolpert on the Weil-Petersson geometry.

• **Damien Gayet** (Institut Fourier Grenoble)

Universal representatives of the homology of complex projective hypersurfaces

Abstract. Let $\Sigma \subset \mathbb{R}^n$ be a connected smooth compact hypersurface with non-vanishing Euler characteristic (which implies that n is odd). I will explain that for any d large enough, the homology of any degree d complex hypersurface of $\mathbb{CP}^n$ possesses a basis such that a uniform positive proportion of its members can be represented by a submanifold diffeomorphic to Σ.
Quite surprisingly, the proof is of probabilistic nature.

• **Henri Guenancia** (Institut de Mathématiques de Toulouse)

A Miyaoka-Yau inequality for minimal models

Abstract. I will explain a joint work with Behrouz Taji where we provide a inequality between Chern classes on a complex projective variety X with klt singularities and nef canonical bundle. The inequality bridges a gap between celebrated inequalities due to Miyaoka (when X is not uniruled) and Yau (when X is smooth with ample canonical bundle). If time permits, I will also mention a generalization of the inequality in the context of log pairs (X, D).

• **Jérémy Guéré** (Institut Fourier Grenoble)

Gromov-Witten theory for orbifold hypersurfaces

Abstract. In this talk, I will first motivate Gromov-Witten theory as providing a deformation of the cup product, known as quantum cohomology. I will then explain what the main obstruction to its computation is, namely the non-vanishing of a cohomology group called non-convexity. Eventually, I will state my recent results obtained for hypersurfaces in weighted projective spaces and my program to compute Gromov-Witten invariants in more general situations using a theorem of Costello relating invariants of a projective variety to those of its symmetric products.

• **Grégoire Menet** (Institut Fourier Grenoble)

On compact hyperkähler orbifolds

Abstract. Since Bogomolov's decomposition theorem, hyperkähler manifolds play an important role in algebraic geometry; they can be considered as elementary bricks for classifying kähler manifolds with trivial first Chern class. However, in the framework of the minimal model program, we remark that considering only smooth varieties is not enough to provide a satisfactory classification. Hyperkähler orbifolds partially answer to this problem. An orbifold is a generalization of a manifold obtained by gluing quotients of open sets of $\mathbb{C}^n$ by finite groups. In this talk, I will give an overview of the recent progress in this area and sketch a classification of the known examples.

• **Philipp Naumann** (Universität Bayreuth)

Curvature of higher direct images for log canonically polarized families

Abstract. For a family of log canonically polarized pairs, the bundle of relative logarithmic p-forms with values in the relative canonical bundle has a singular hermitian metric that is induced by the complete Kähler-Einstein metrics on the open fibers. By using L^2 theory, we are able to equip the higher direct images of this bundle with a smooth L^2 metric, whose curvature can be computed. As an application, we prove the hyperbolicity of the moduli space of log canonically polarized manifolds in the orbifold sense.

• **Dan Popovici** (Institut de Mathématiques de Toulouse)

Some Aspects of Higher-Page Non-Kähler Hodge Theory

Abstract. This is joint work with Jonas Stelzig and Luis Ugarte. Its main thrust is to extend some basic results in Hodge Theory to the higher pages of the Frölicher spectral sequence. For an arbitrary nonnegative integer r, we introduce the class of page-r-$\partial\bar{\partial}$-manifolds by requiring the analogue of the Hodge decomposition to hold on a compact complex manifold X when the usual Dolbeault cohomology groups $H^{p,\,q}_{\bar{\partial}}(X)$ are replaced by the spaces $E^{p,\,q}_{r+1}(X)$ featuring on the $(r+1)$-st page of the Frölicher spectral sequence of X. The class of page-r-$\partial\bar{\partial}$-manifolds increases as r increases and coincides with the usual class of $\partial\bar{\partial}$-manifolds when $r = 0$. We investigate various properties of these manifolds and show that they are analogous to those of $\partial\bar{\partial}$-manifolds with some noteworthy exceptions. We also point out a number of examples. For instance, all complex parallelisable nilmanifolds, including the Iwasawa manifold and a 5-dimensional analogue thereof, are page-1-$\partial\bar{\partial}$-manifolds, although they are seldom $\partial\bar{\partial}$-manifolds. The deformation properties of page-1-$\partial\bar{\partial}$-manifolds are also investigated and a general notion of essential small deformations is introduced for Calabi-Yau manifolds. We also introduce higher-page analogues of the Bott-Chern and Aeppli cohomologies and highlight their relations to the new class of manifolds. On the other hand, we prove analogues of the Serre duality for the spaces featuring in the Frölicher spectral sequence and for the higher-page Bott-Chern and Aeppli cohomologies.

• **Sheng Rao** (Wuhan University & Institut Fourier Grenoble)

Geometry of logarithmic forms and deformations of complex structures

Abstract. We present a new method to solve certain dbar-equations for logarithmic differential forms by using harmonic integral theory for currents on Kahler manifolds. As applications, we generalize the result of Deligne about closedness of logarithmic forms,

give geometric and simpler proofs of Deligne's degeneracy theorem for the logarithmic Hodge to de Rham spectral sequences at E1-level, as well as certain injectivity theorem on compact Kahler manifolds.

Furthermore, for a family of logarithmic deformations of complex structures on Kahler manifolds, we construct the extension for any logarithmic (n,q)-form on the central fiber and thus deduce the local stability of log Calabi-Yau structure by extending an iteration method to the logarithmic forms. Finally we prove the unobstructedness of the deformations of a log Calabi-Yau pair and a pair on a Calabi-Yau manifold by differential geometric method.

This talk is based on a recent joint work with Kefeng Liu and Xueyuan Wan.

• **Xiaojun Wu** (Institut Fourier Grenoble)

Pseudo-effective reflexive sheaves with vanishing first Chern class

In the papers of Campana-Cao-Matsumura and Hosono-Iwai-Matsumura, it is proven that a pseudo-effective reflexive sheaf with vanishing first Chern class over a projective manifold is in fact a nef vector bundle. In this talk we will talk about the generalisation of this result to the compact Kähler case. The proof uses another approach of Segre currents (positive currents in the Segre class). A similar construction of Segre currents can be found in Lärkäng-Raufi-Ruppenthal-Sera.

• **Mikhail Zaidenberg** (Institut Fourier Grenoble)

High transitivity in algebra and geometry

Abstract. An infinite group G is called *highly transitive* if it acts on some infinite set m-transitively for any natural number m. We give a brief survey on some recent results on abstract highly transitive groups.
Then we pass to examples of affine algebraic varieties with the automorphism group acting highly transitively. Specifically, for toric affine varieties a highly transitive group can be generated by a finite number of one-parameter subgroups; for the affine spaces, three such subgroups suffice. We formulate some open problems related to group growth, and explain some partial results.